Transformer Coupling Circuits for High-Frequency Amplifiers

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This article deals with the use of transformer type of coupling circuits in high-frequency amplifiers to transmit efficiently voltages or currents between certain limiting frequencies while attenuating those above and below the limiting frequencies. The similarity of these coupling circuits to band-pass filters is shown and the conditions to be satisfied in order that they may act as such are covered. Means of obtaining uniformly high amplification over relatively wide frequency bands are explained. Typical conditions under which these coupling circuits have been employed and factors affecting their performance are discussed.

I. Introduction

THE designer of high-frequency amplifiers is often confronted with the problem of obtaining, with a given number of amplifying tubes and coupling circuits in cascade, maximum voltage amplification over a predetermined band of frequencies, and high attenuation to all voltages outside of the desired band of frequencies. To secure a large voltage amplification the most convenient, economical and practical arrangement for coupling the various stages of the amplifier is by means of the step-up transformer. By adding condensers in parallel with one or more of the windings of the transformer a frequency discrimination characteristic is obtained which can be controlled to a large extent by the proper choice of the transformer constants and the tuning capacitances. In the case of the usual type of transformer coupling with the secondary winding tuned to resonance at a given frequency the voltage amplification for a single stage depends on the resistance of the secondary winding, the conductance of the grid circuit of the second tube and the size of the tuning condenser. With proper choice of the transformer constants very large amplification can generally be obtained over a relatively narrow band of frequencies. To obtain high amplifications over wider frequency bands other factors must be taken into consideration.

Possibly the most important of these factors is the impedance of the circuit into which the secondary winding operates. In the case of an interstage transformer with the secondary winding connected directly to the grid circuit of a three-element tube this impedance depends on the electrode capacities of this tube, the amplification factor, the plate impedance and the impedance connected to the output terminals of the tube. Where the amplification factor and plate circuit terminating impedance are low, as in the case of tubes operating as demodulators, the effective input impedance of the tube is comparatively high and depends mainly on the electrode capacities. The input impedance of a shielded-grid tube is likewise comparatively high and depends mainly on the capacity between the grid and filament terminals. The input impedance of a tube decreases with increase in frequency and at very high frequencies it becomes so low that difficulty is experienced in obtaining any increase in amplification in the coupling circuit. In the case of the single tuned transformer, previously referred to, the effective input capacity of the tube may act as part of the tuning condenser with the result that at the resonance frequency a substantial amplification can usually be obtained. However, when it is desired to transmit a broad band of frequencies the magnitude of the effective input capacity becomes a very important controlling factor. For a definite band width the possible voltage amplification varies inversely as the value of the effective capacity across the secondary winding.

A study of the various factors entering into the use of a suitable coupling circuit for successive stages of a high frequency amplifier has indicated that best results are obtained if the design of the transformer is based upon the principles of the broad-band filter. Definite relations are obtained between the constants of the transformer windings, the tuning capacities, and the impedances of the circuits between which it operates. The structure is thus essentially a band-pass filter and has all of its elements properly proportioned to provide the desired band selectivity, but it still retains the form and the functions of a transformer.

II. Theory and Method

Fig. 1 shows a simple transformer type of coupling circuit connected between the plate circuit of an amplifying tube and the input circuit of a second tube. The primary and secondary circuits of the trans-

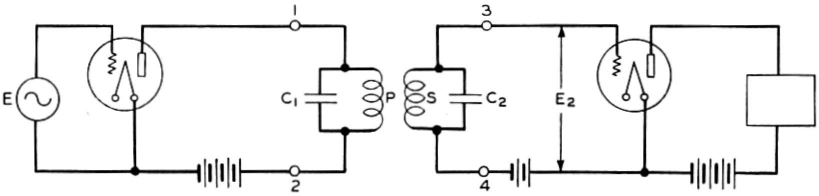

Fig. 1—Circuit schematic of an amplifier using a transformer type of coupling circuit.

former are adjusted to resonate at the same frequency. The determination of the constants for the circuit so that it conforms not only to the requirements relating to the band selectivity and to the voltage transformation ratio, but also to requirements of transmission efficiency, is explained in the following mathematical analysis.

For convenience in analyzing the essential parts of Fig. 1 they are shown in a simplified schematic form in Fig. 2. In this schematic, the sending-end impedance Z_0 corresponds to the effective plate impedance of the first vacuum tube and the electromotive force E_1 corresponds to the internal plate voltage of this tube. The transformer formed by inductances P and S is replaced by the well known "T" network.[1] The capacity C_1 is provided by a separate condenser, while that of C_2 is equal to the effective distributed capacity of winding S plus the effective input capacity of the second vacuum tube.

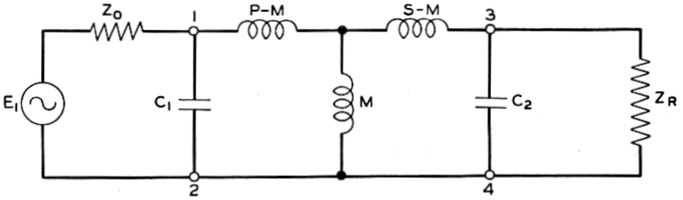

Fig. 2—"T" network of a transformer coupling circuit.

The method used in computing the output voltage of a transmission circuit such as is shown in Fig. 2 consists in determing the "image" impedances corresponding to each pair of terminals. These image impedances are determined from the open and short-circuit impedances, measured at the terminals of the network,[2] by the following relationships:

$$Z_{1,\,2} = \sqrt{Z_0'Z_S'}, \tag{1}$$

and

$$Z_{3,\,4} = \sqrt{Z_0''Z_S''}, \tag{2}$$

where Z_0' and Z_S' equal respectively the open and short-circuit impedances and $Z_{1,\,2}$ the image impedance at terminals 1, 2 and Z_0'' and Z_S'' equal the open and short-circuit impedances and $Z_{3,\,4}$ the image impedance at terminals 3, 4.

Since the resistance elements of a transformer of this type can be made very small in comparison to the reactance elements, it is practical to eliminate them in computing the open and short-circuit impedances.

[1] "Telephone Transformers," by W. L. Casper, *A. I. E. E. Jour.*, March, 1924, Vol. XLIII, No. 3.

[2] "Transmission Characteristics of Electrical Wave Filters," by O. J. Zobel, *Bell Sys. Tech. Jour.*, October, 1924, Vol. III, No. 4.

With the assumption that the elements of the coupling circuit are pure reactances and that the two anti-resonant circuits are at resonance at the same frequency, f_0, the following expression for $Z_{1,\,2}$ as a function of frequency f was derived with the use of equations 1 and 2.

$$Z_{1,\,2} = \frac{1}{2\pi f C_1}\sqrt{\frac{1 - K^2}{\left(1 - K - \frac{f_0^2}{f^2}\right)\left(\frac{f_0^2}{f^2} - 1 - K\right)}} \tag{3}$$

and $Z_{3,\,4}$ is the same expression with C_2 substituted for C_1 or

$$Z_{3,\,4} = Z_{1,\,2} \times \frac{C_1}{C_2} = Z_{1,\,2}\frac{S}{P},$$

where K is the coefficient of coupling between inductances P and S, and f_0 is the common resonance frequency of the two anti-resonant circuits in Fig. 1.

The lower and upper cut-off frequencies f_1 and f_2 are related to the resonance frequency f_0 as follows:

$$\begin{aligned} f_0 &= f_1\sqrt{1 + K} \\ &= f_2\sqrt{1 - K}. \end{aligned} \tag{4}$$

The geometrical mean frequency

$$\begin{aligned} f_m &= \sqrt{f_1 f_2} \\ &= \frac{f_0}{\sqrt[4]{1 - K^2}}. \end{aligned}$$

Substituting the above expressions for f_m in place of f in equation (3), we find that at the geometrical mean frequency the image impedance in terms of f_1 and f_2 has the value

$$\frac{1}{2\pi(f_2 - f_1)C_1}, \tag{5}$$

which will be denoted by $Z_{1,\,2}$.

It will be noted upon examining equation (3) that $Z_{1,\,2}$ and $Z_{3,\,4}$ are resistive between f_1 and f_2 and are minimum at the geometrical mean frequency. They increase uniformly on either side of the geometrical mean frequency and are infinite at f_1 and f_2. For frequencies below f_1 and above f_2 they are reactive. It can therefore be seen that this type of coupling circuit inherently possesses the characteristics of a band-pass filter. However, in order that the band characteristic may be properly developed, the magnitudes of the various elements must be proportioned with respect to the terminal impedances as well as to the limiting frequencies. This is done by making the image impedances about equal to the respective terminal impedances at the mean frequency f_m of the band. Preferably the impedances should be matched at both ends, but if the transformer normally operates with one end substantially open-circuited, as in an amplifier, it is sufficient to effect the matching at the other end. The following formulas giving the constants of the various elements in terms of the limiting frequencies were derived from the foregoing equations.

$$K = \frac{f_2^2 - f_1^2}{f_2^2 + f_1^2}, \tag{6}$$

$$f_0 = \frac{\sqrt{2} f_1 f_2}{\sqrt{f_1^2 + f_2^2}}, \tag{7}$$

$$C_1 = \frac{1}{2\pi(f_2 - f_1)\, Z_{1,\,2}}, \tag{8}$$

$$P = \frac{1}{4\pi^2 f_0^2 C_1}, \tag{9}$$

$$S = \frac{1}{4\pi^2 f_0^2 C_2}, \tag{10}$$

or

$$S = P \times \frac{C_1}{C_2}. \tag{11}$$

As previously mentioned the above equations are based on the ideal condition which assumes the elements to be pure reactances. Actually, a small amount of resistance is present in each element of the transformer which tends to reduce the width of the transmission band. Consequently f_1 and f_2 should be assumed slightly lower and higher respectively than the lower and upper frequencies of the desired transmitted band in order to insure uniform voltage amplification. It is advantageous to make the impedance $Z_{1,\,2}$ approximately 0.8 of the terminal impedance Z_0, in which case $Z_{1,\,2}$ and Z_0 will be equal at two frequencies near the band limits and will not be greatly mismatched at the geometrical mean frequency. This tends to improve the uniformity of transmission within the band.

In applying the above equations, C_1 is determined first from equation (8). P is then determined from equation (9). From a knowledge of the effective distributed capacity of S and the input capacity of the second tube in Fig. 1, S is determined from equation (10). The windings P and S are then arranged with respect to each other to satisfy the coupling coefficient of equation (6). To facilitate adjustment C_2 may include a small adjustable condenser to compensate for variations in winding capacities as well as in the input capacity of the vacuum tube.

The following analysis shows that maximum voltage amplification is obtained with practically uniform transmission for all frequencies within the transmitted band when the secondary circuit is terminated only in its tuning condenser.

Referring to Fig. 2 and assuming that $Z_{1,\,2} = Z_0$ and $Z_R = Z_{3,\,4}$, we find that the voltage drop across Z_R is

$$E_R = \frac{1}{2} E_1 \sqrt{\frac{Z_{3,\,4}}{Z_{1,\,2}}}$$

or

$$E_R = \frac{E\mu}{2} \sqrt{\frac{Z_{3,\,4}}{Z_{1,\,2}}} . \tag{12}$$

Now from Thevenin's Theorem,[3] assuming $Z_0 = Z_{1,\,2}$, we can obtain the following expression for the output current for any value of the impedance Z_R:

$$I_r = \frac{E_2}{Z_R + Z_{3,\,4}} , \tag{13}$$

where E_2 is the open-circuit voltage at terminal 3, 4 and I_r is the current flowing in the terminating impedance Z_R.

The actual voltage across the terminating impedance Z_R is

$$E_R = I_R Z_R = \frac{E_2 Z_R}{Z_R + Z_{3,\,4}} .$$

Where Z_R is equal to $Z_{3,\,4}$ the output voltage has the value

$$E_R = \frac{E_2}{2} .$$

The effect of matching the terminating impedance to the impedance $Z_{3,\,4}$ is thus to cut the output voltage in half.

Then

$$\frac{E_2}{E} = \mu \sqrt{\frac{Z_{3,\,4}}{Z_{1,\,2}}} = \mu \sqrt{\frac{C_1}{C_2}} , \tag{14}$$

where μ = voltage amplification constant of the first tube.

Generally the effect of leaving the output end of a filter open-

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[3] "Transmission Circuits for Telephone Communication," by K. S. Johnson, p. 79.

circuited would be to introduce irregularities of transmission within the band. However, if the band is relatively narrow the resistances of the filter elements are sufficient to smooth out these irregularities and in certain cases the transmission may be made more uniform by the omission of the impedance Z_R. This has been found to be the case with transformers of the type described here.

Equation (14) shows that the voltage ratio E_2/E which is the ratio of the open-circuit voltage of the tuned circuit at terminals 3, 4 to the applied grid voltage of the first tube is directly proportional to $\sqrt{\frac{C_1}{C_2}}$ over the transmitted band. By proper choice of the constants of the coupling circuit so that $Z_{1,\,2}$ will equal Z_0 at a frequency lower and higher than the geometrical mean frequency this voltage ratio will remain practically constant for all frequencies within the band.

The voltage ratio E_2/E for any frequency ω outside the transmission band may be obtained from the following equation:

$$E_2/E = u\left[\frac{Z_1}{Z_2} \times \frac{Z_3}{Z_4} \times Z_5/Z_6\right],$$

where

$$Z_1 = \frac{1}{j\omega C_2}; \qquad Z_2 = j\omega(S - M) + Z_1; \qquad Z_3 = \frac{Z_2 j\omega M}{Z_2 + j\omega M};$$

$$Z_4 = Z_3 + j\omega(P - M); \qquad Z_5 = \frac{Z_4 \dfrac{1}{j\omega C_1}}{Z_4 + \dfrac{1}{j\omega C_1}}; \qquad Z_6 = Z_5 + Z_0.$$

P, S and M are in henrys and C_1 and C_2 in farads.

As the effective voltage amplification of the network within the transmission band is directly proportional to

$$\sqrt{\frac{C_1}{C_2}}$$

it is evident that since C_1 is fixed by equation (8), the value of C_2 must be kept to a minimum to realize maximum voltage step-up. Consequently, greater amplification will be obtained if the second tube in Fig. 1 has minimum input capacitance. The effective distributed capacitance of the winding is kept at a low value by placing the winding in narrow grooves on a spool.

In actual use the maximum amplification realized at the higher frequencies is somewhat less than that shown in equation (14) since the conductance component of the input impedance of the second tube reduces the effective voltage at this point. It is therefore of importance that the conductance component as well as the capacitance component of the input impedance of the second tube be kept to a minimum if maximum amplification is to be obtained. If the second tube in Fig. 1 is operated as a negative grid bias detector the ratio of E_2/E may be made considerably larger than if it is operated as an amplifier. The application of a large negative grid bias and the use of a by-pass condenser in the plate circuit of the tube to improve its modulation efficiency usually results in a very small input capacitance and conductance. It is therefore sometimes desirable in multistage amplifiers to place the selective circuits between the last amplifier tube and the detector tube. With the advent of the shielded-grid tube having the characteristic of low input capacitance and conductance, transformers of this type are particularly adapted to high frequency amplifiers such as used for the intermediate frequency amplifier of a superheterodyne receiver.

III. Practical Applications

Fig. 3 shows the transmission characteristic of a simple transformer coupling circuit when operating from a balanced resistance of 1000 ohms into the grid circuits of push-pull amplifying tubes. Stability

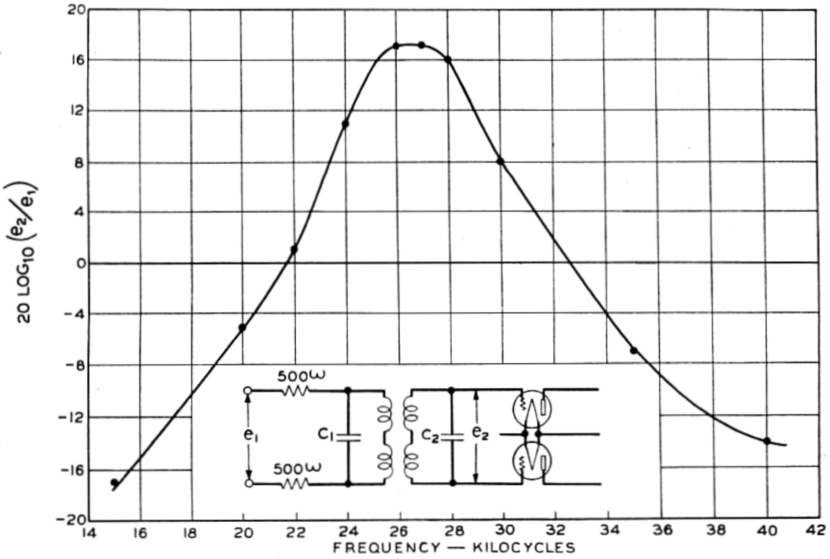

Fig. 3—Typical transmission characteristic of a transformer coupling circuit.

was more important in this particular case than maximum voltage step-up so that a fixed condenser C_2 was added externally across the grid circuit to prevent the operation of the transformer from being affected by variations in the grid circuit capacities.

More than one coupling circuit has been used to obtain the necessary selectivity. Several of these have been connected together with either series or shunt condensers to obtain the equivalent of a multi-section band-pass filter. Networks of this type in which the elements have been chosen in accordance with the previously mentioned equations have been employed as band filters because of their simplicity and compactness, and the relatively low cost of the inductance elements. They have been used to connect two equal or unequal impedances as well as to operate from an impedance directly into the grid circuit of a vacuum tube.

One of these networks was used between the plate circuit of a shielded-grid vacuum tube and the grid circuit of a second shielded-grid vacuum tube at 84 kilocycles. The gain characteristic of a stage of this type is shown in Fig. 4. The capacitance C_1 represents the capacitance between the plate and shield of the first tube plus the winding capacitance and capacitance C_2 represents the winding capacitance and effective input capacitance of the second tube. It will be of interest to know that the voltage amplification obtained over the transmitted band was approximately equal to the amplification of the first shielded-grid tube. Consequently, the transformer network

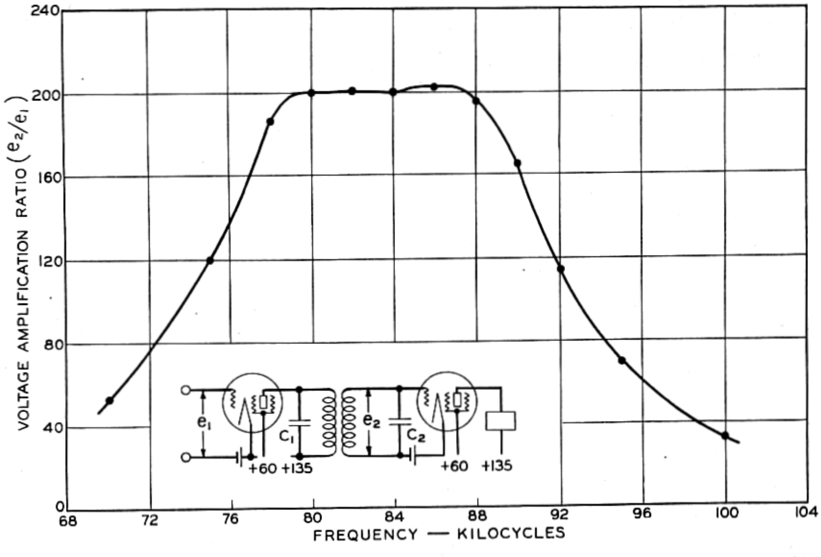

Fig. 4—Transmission characteristic of a transformer coupling circuit operating between shielded-grid tubes.

efficiently transmitted the internal plate voltage of the first tube to the grid circuit of the second tube. The internal plate impedance of the particular tube used was approximately 400,000 ohms. A two-stage amplifier consisting of two shielded-grid amplifying tubes and two transformer coupling circuits connected in cascade gave a voltage amplification of approximately 40,000 times over the frequency band shown.

Another type of transformer coupling circuit which was employed in the intermediate frequency amplifier of a high quality superheterodyne radio receiver is shown in Fig. 5. The circuit schematic of this transformer and its transmission characteristic are shown in Fig. 6. It will be noted that two transformers are mounted separately and electrically connected by a series condenser C_2'. C_1 and L_1 were determined from equations (8) and (9) and $L_1 = L_2 = L_3$. $C_2' = \frac{C_1}{2}$. The capacitance across L_4 was equal to the winding capacity and effective input capacity of the second tube. The second tube of Fig. 6 was the second detector of the intermediate frequency amplifier.

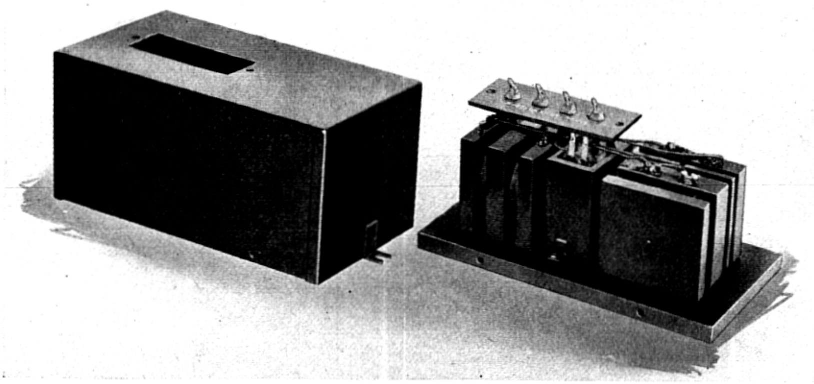

Fig. 5—Coupling circuit consisting of two-tuned transformers connected in cascade.

The elements of a coupling circuit consisting of three transformers and their associated condensers for operation over the carrier frequency range of 50 to 150 kilocycles are shown in Fig. 7. The windings of stranded wire are applied in narrow grooves to reduce the dielectric losses of the insulation between layers. The ratio of the reactance to the effective resistance for these coils varies from approximately 150 at 50 kilocycles to approximately 240 at 150 kilocycles. The halves of the winding connected to the grid circuit of the balanced tubes

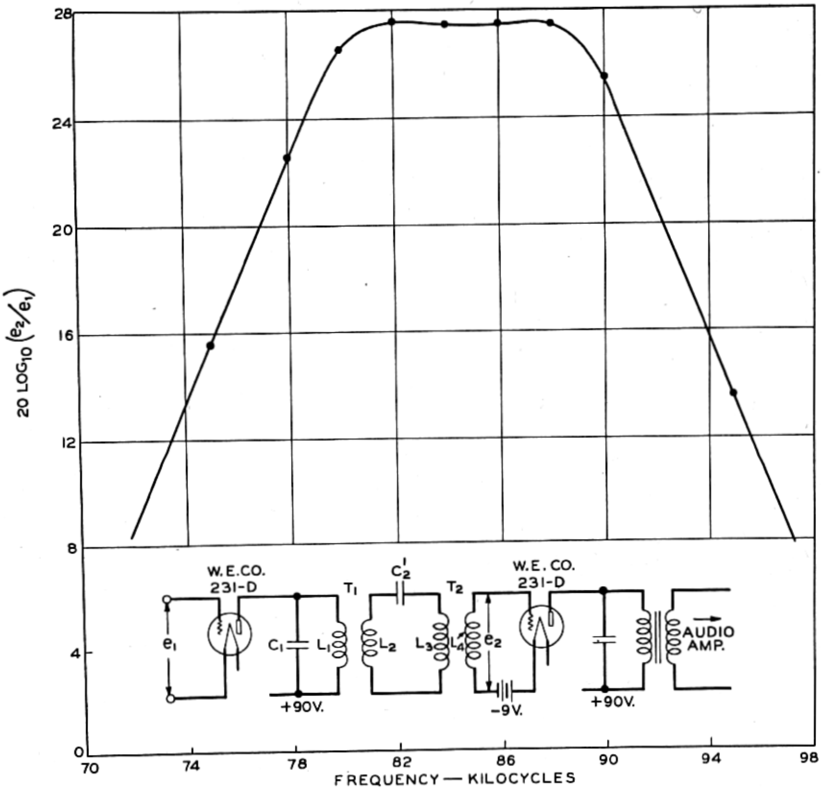

Fig. 6—Transmission characteristic of a double transformer type of tuned circuit.

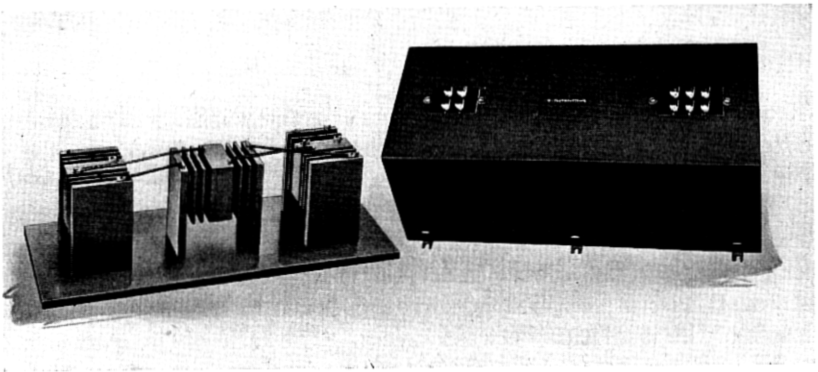

Fig. 7—Coupling circuit consisting of three-tuned transformers in cascade.

were wound in the two grooves of the last spool type assembly in such a manner as to maintain satisfactory balance for this type of operation. Four adjustable condensers were added externally to these transformers as shown in Figs. 8 and 9. The capacity of the condensers was adjustable so that the transmitted band could be located anywhere between 50 and 150 kilocycles.

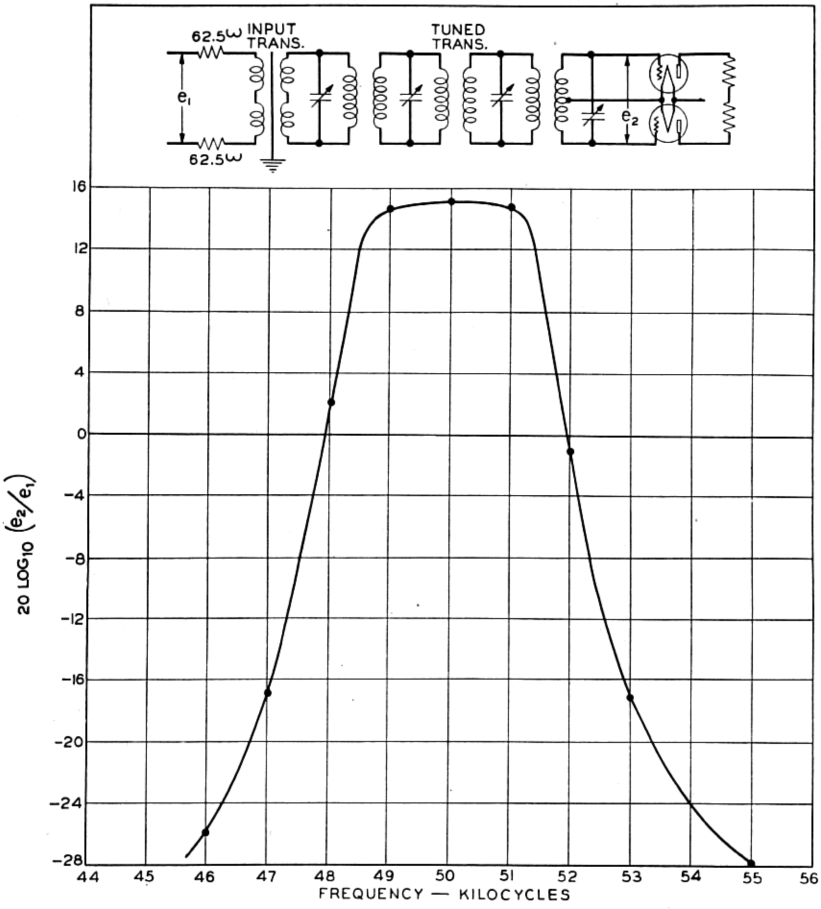

Fig. 8—Transmission characteristic of a triple transformer type of tuned circuit.

Fig. 8 shows the circuit schematic and transmission characteristic of the same transformer network at 50 kilocycles when operating from a non-inductive resistance into the grid circuit of push-pull modulator tubes. A step-up transformer was used between the resistance and the tuned transformer to match impedances and improve the efficiency of transmission. Fig. 9 shows the transmission characteristic and circuit schematic of the same tuned transformer when operating between two non-inductive resistances. Repeating coils were used

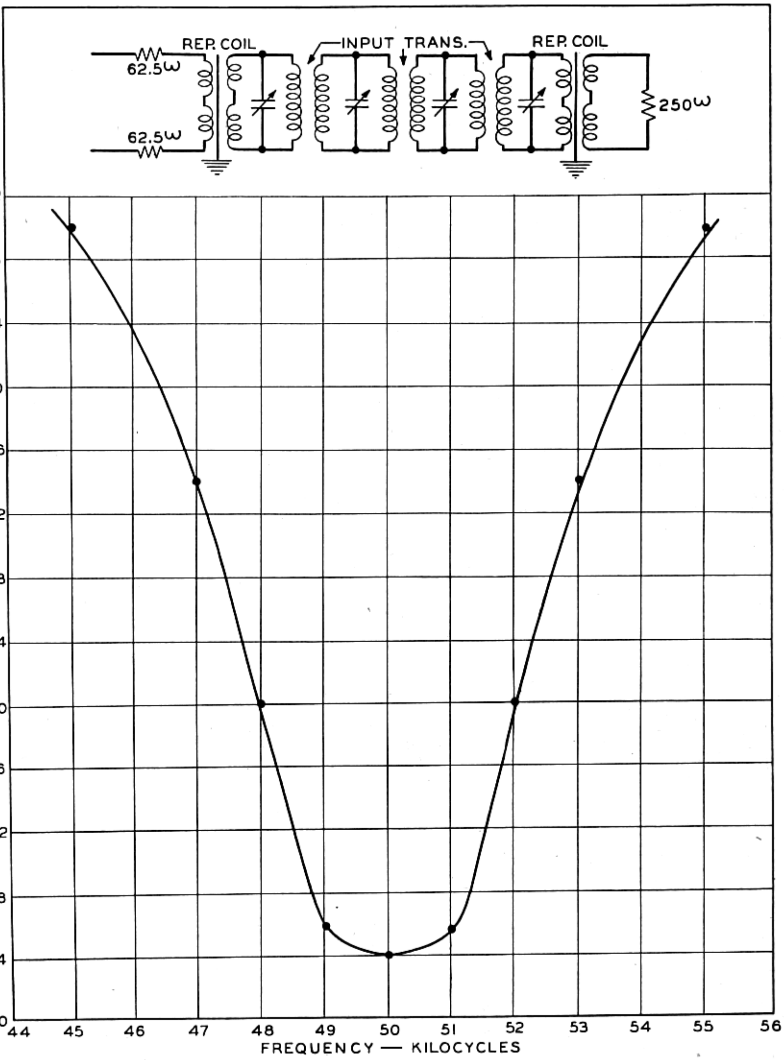

Fig. 9—Transmission loss characteristic of a triple transformer type of tuned circuit.

between the transformer and the terminated resistances to match impedances. Although the width of the transmission band at 50 kilocycles is only 2500 cycles it is considerably wider at 150 kilocycles as one would expect from the equation showing the relation of band width and capacity. This, however, was not an objectionable feature in the circuit in which the transformer was employed. In order to maintain a constant band width irrespective of its location, the capacity must remain constant and the self impedance of the windings and the coupling coefficients changed or the inductance elements maintained at a constant value and the capacities and coupling coefficients changed.

IV. Conclusions

Transformer types of coupling circuits having the inductive and capacitive elements proportioned as explained in this paper are essentially band-pass filters and are therefore particularly adapted to high-frequency amplifiers or circuits where it is necessary to transmit efficiently frequencies within a desired band while strongly attenuating all frequencies outside the band. By proper choice of the transformer constants and the condensers it has been shown that uniformly high voltage amplification was obtained over relatively wide frequency bands. It has also been shown that maximum uniform voltage amplification for a given frequency band was obtained when the output terminals of a transformer coupling circuit were terminated only in a condenser.

A few applications of transformer coupling circuits have been discussed and the individual characteristic shown. It should be understood, however, that these coupling circuits are not limited to the frequency bands illustrated but may be efficiently used at higher frequencies and over wider transmission bands.

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