Programme

Receptivity of a Longitudinal Contra-Rotating Vortex Pair in an External Flow: A Numerical Experimentation

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1. Introduction

The problem developed in this contribution is encountered in airplane aerodynamics and concerns the influence of long life longitudinal wake vortices generated by wing tips or by external obstacles such as reactors or landing gears. More generally it concerns 3D bodies of finite extension in cross flow. At the edge of such obstacles, longitudinal vortices are created by pressure differences inside the boundary layers and rotate in opposite senses. The behavior of these vortices has been studied in great detail by many authors as reported in [1]. The numerical simulations are time evolution computations on a fixed space box with periodic boundary conditions. In the present work perturbations of the basis flow are analyzed in order to describe the receptivity mechanism to the periodic Crow instability [2] and to random or turbulent perturbations.

2. Basis Flow

The initial conditions for the simulation are the analytical solutions of an array of periodic vortices with images satisfying the different boundary conditions. The element of this array is a dipole of Oseen vortices of opposite circulation +/- Γ_0 separated by a distance b_c. The rotation velocity $u_\theta(r)$ of each vortex in cylindrical coordinates is given by equation (1) as well as the transverse block displacement w_d and the Reynolds number Re:

$$u_\theta(r) = \frac{\Gamma_0}{2\pi r}\left(1 - e^{-r^2/r_0^2}\right) \; ; \; U_{ref} = w_d = \frac{\Gamma_0}{2\pi b_c} \; ; \; \mathrm{Re} = \frac{\Gamma_0}{2\pi\nu} . \qquad (1)$$

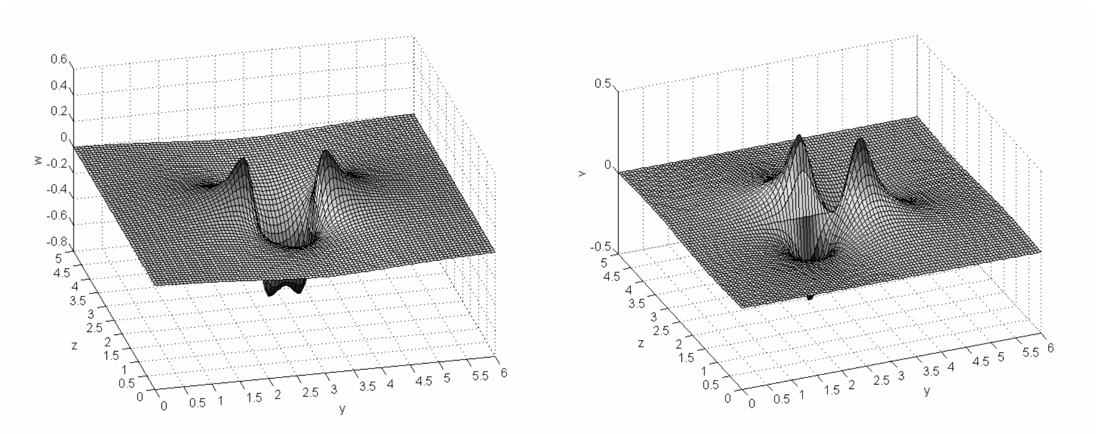

Fig.1. Distribution of the initial velocity field for a vortex dipole

3. Numerical simulation

The system of incompressible and unsteady Navier-Stokes equations is solved on a 3D orthogonal Cartesian grid by using the numerical code JADIM, developed at IMFT by Calmet and Magnaudet [3] and extended by Péneau et al. to boundary layer flows [4] and free-stream turbulence effects [5]. In this code a dynamical mixed model is used in order to introduce energy backscatter from small structures to larger ones. Such a mechanism is likely to occur in transitional flows and in frontier zones between large and small scales of turbulence. It is believed that this is the case near the core of the vortices. The use of this boundary layer code is made in the goal of further studying the vortex pair with spatial evolution in a turbulent external mean flow of non zero mean longitudinal velocity.

For modeling a longitudinal contra-rotating vortex pair we use a rectangular calculation domain as presented in fig. 2.

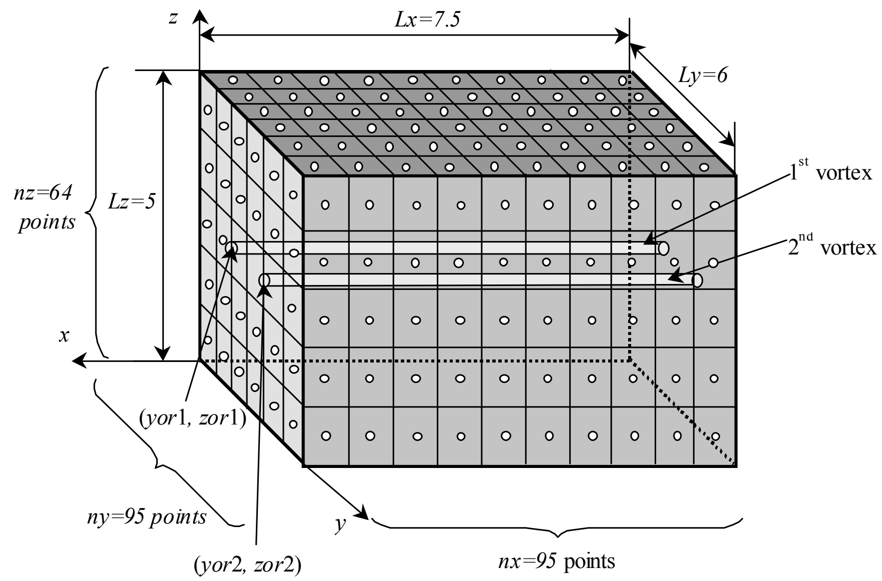

Fig.2. Grid parameters

The grid parameters of the calculation domain give 577 600 computation cells. The boundary conditions used are periodicity in Oz and Ox axes, and symmetry in Oy axes. These conditions enable to describe the temporal evolution of the studied phenomenon.

4. Perturbation imposed to the basis flow

The first studied perturbation is periodic in space and time. It is the long wavelength Crow instability [4]. This perturbation is obtained by disturbing the position of the vortex centers from basis flow situation as shown of figure 3 and detailed in equation (2).

$$\begin{aligned} y_{c\_pert} &= y_{c\_nepert} + Amp \cdot \sin(kx + \varphi) \cdot \cos\theta \\ z_{c\_pert} &= z_{c\_nepert} + Amp \cdot \cos(kx + \varphi) \cdot \sin\theta \end{aligned} \qquad (2)$$

where $(y_{c\_nepert}, z_{c\_nepert})$ is the position of unperturbed vortices, Amp is the amplitude of disturbance, k is the longitudinal wave number , φ is the phase, and θ is the disturbance plane angle.

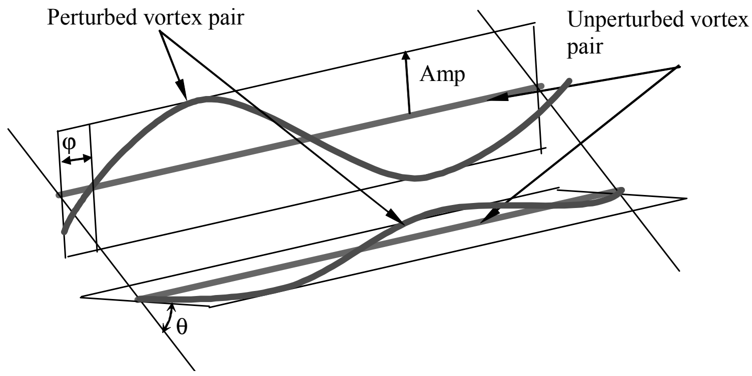

Fig.3. Displacement of vortex cores in the Crow perturbation

After perturbing initially the basis flow, a fixed number of iterations are performed in order to eliminate errors due to the disturbances. The pressure and velocity fields obtained constitute the initial condition for starting the physically relevant simulation.

The other perturbations are random and they differ by space and time correlation lengths. We examine both a white noise and a synthetic turbulent field. The white noise intensity is fixed as an arbitrary fluctuation superimposed on the local velocity of the basis flow:

$$u = u_0(1+u')\ ;\ v = v_0(1+v')\ ;\ w = w_0(1+w'), \tag{3}$$

where (u', v', w') are random numbers with uniform distribution in the range $[-Amp, Amp]$ with $Amp = u_{\theta\max} \cdot 10^{-5}$, noise amplitude. The white noise is injected continuously after each iteration. Because the values (u', v', w') are randomly generated they do not satisfy the continuity equation. This situation can induce noticeable errors but the disturbances entered are small and it is assumed that they do not significantly break the continuity constraint.

An important objective of this study consists in depicting the effect of an external turbulent field on a pair of longitudinal contra-rotating vortices. For that we achieved a procedure of generating a turbulent field issued from the interaction between of Oseen vortices with radius $R \in [R_{min}, R_{max}]$ and circulations $\Gamma \in [-\Gamma_{max}, \Gamma_{max}]$ generated randomly. The orientations, positions of vortex centers and lengths are also randomly distributed. The intensity of resulting turbulent fields may be controlled through the intensity, length, maximal ray and number of elementary vortices. The characteristic parameters used for generating the field of external turbulence are: vortices' radius $R \in [0.01,\ 0.1]$, intensity $\Gamma \in [-0.01,\ 0.01]$ and length $L = [2,\ 8]$ cells.

The generated vorticity field is used as initial condition for a pre-computation stage. After a fixed number of temporal iterations (over 500) in the LES code, the velocity and pressure fields represent the field of external turbulence that is applied to the pair of longitudinal contra-rotating vortices. An instantaneous view of local vorticity fields in the generation box is represented on figure 4.

5. Results

We have first tested that the laminar solution starting from the analytical flow field is numerically stable. The diffusion process is weak and both the maximum vorticity and the effective vortex radii undergo small changes for computational times long compared to the time scales of the perturbations introduced for forcing the initial flow.

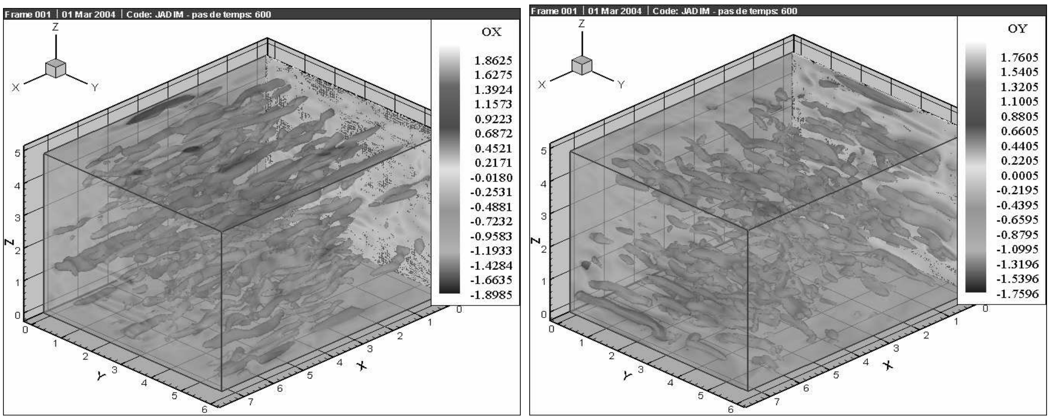

Fig.4. Instantaneous distribution of ω_x and ω_y for the external turbulence

a) Periodic Crow instability

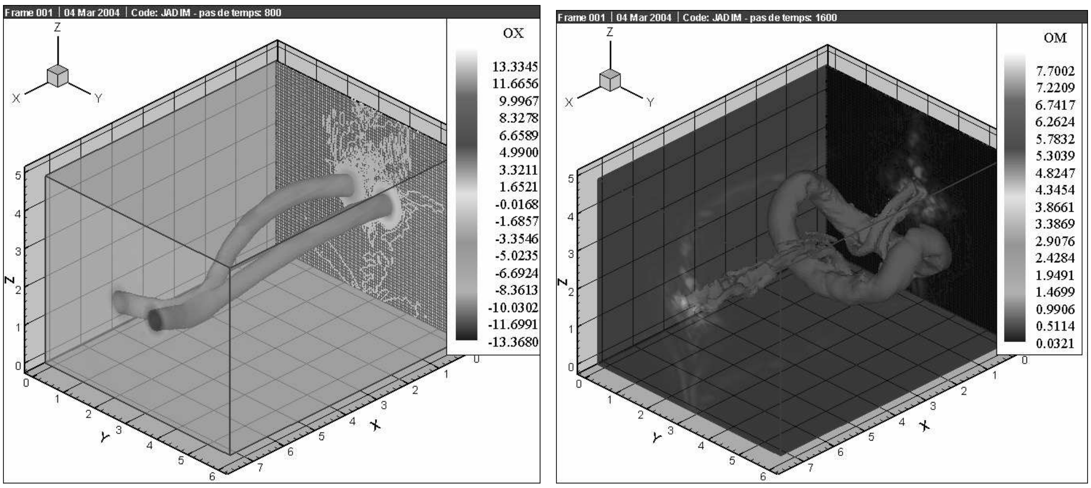

Fig. 5- Iso – ω_x surface in Crow instability development at T=8 and T=16

The instability mechanism induces large oscillations that result in vortex rings. This mechanism is known to be faster to modify the vortex pair than the high wave number elliptic instability. The results of this study are comparable to the ones of literature reported in [1].

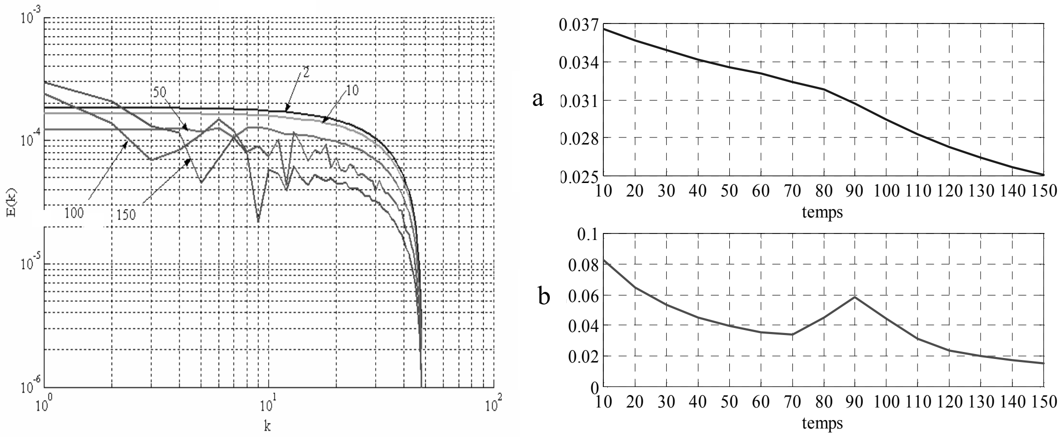

Fig. 6 – Energy spectrum and overall fluctuating energy (a) and enstrophy (b) (Effect of white noise)

During the phase of the long wave instability the vortex tubes come close to each other then the reconnection mechanism takes place between the two tubes. During that period the phenomenon of bounding occurs between the non connected tails of the tube along with the creation of threads with the remanents of the vortex tubes. It can be shown that enstrophy increases during the reconnection. But after reaching a local maximum both kinetic energy and enstrophy are destroyed by viscous dissipation of the small scales.

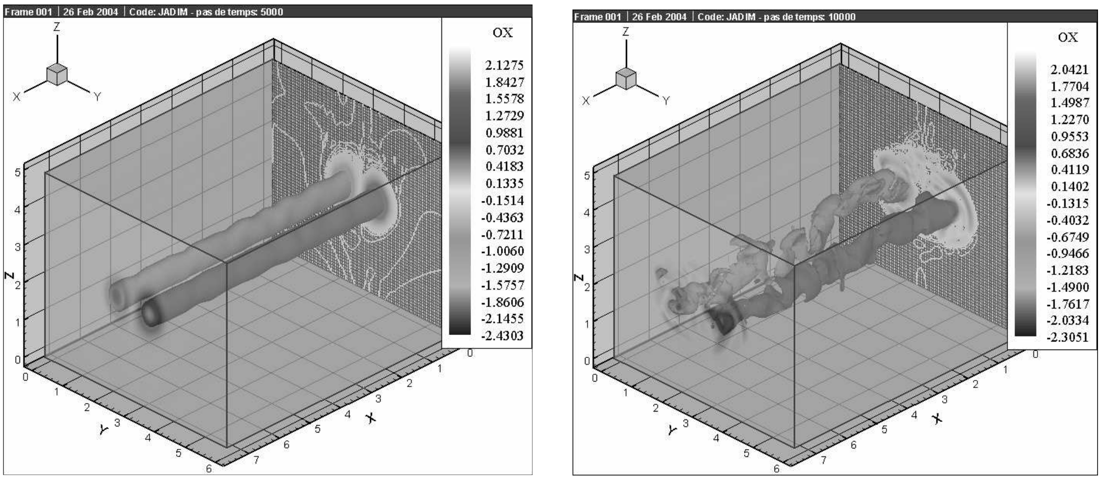

Fig.7. Iso-longitudinal vorticity ω_x at T=50 and T=100 (effect of white noise)

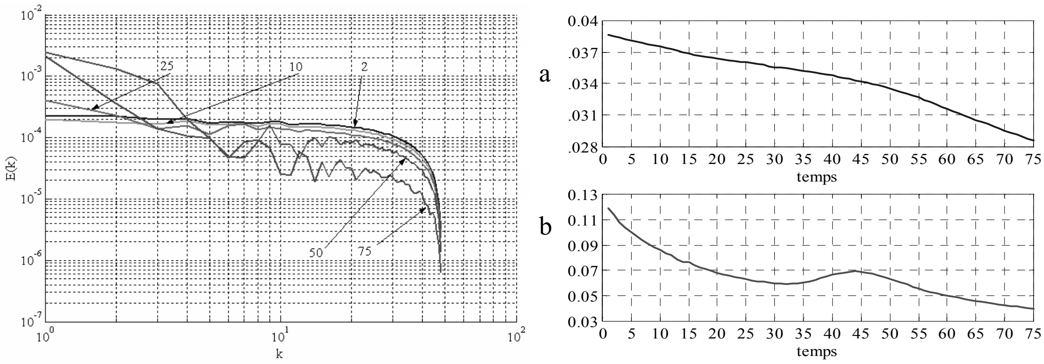

Fig.8. Energy spectra and overall fluctuating energy (a) and enstrophy (b) (effect of external turbulence)

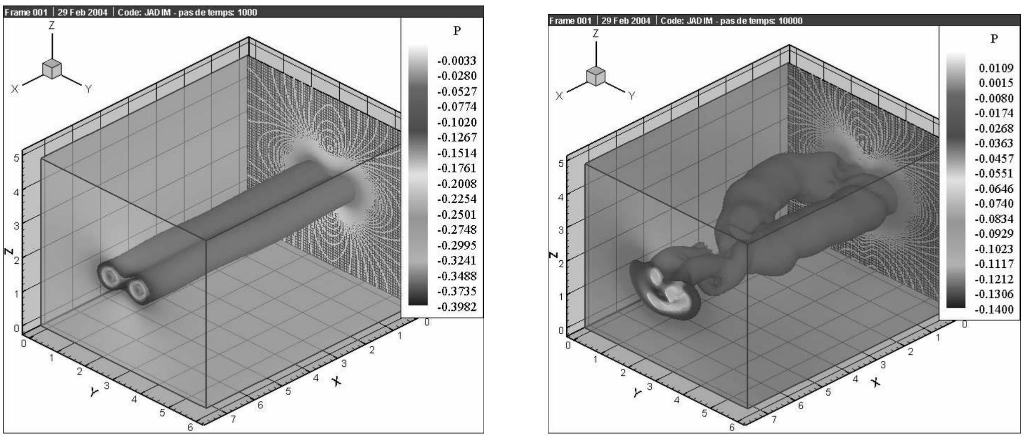

Fig. 9 – Iso - pressure surfaces at time T=50 and 100 (effect of external turbulence)
b) Random white noise

Concerning the white noise level imposed it is shown to be sufficient to trigger the instability and provoke a continuous decrease of energy in time. On figure 6 it can be deduced that preferentially a high wave number mode exhibiting features of elliptical instability is developed at long time (T=100). It is deduced that such a fully random short time and length correlation does produce high wave number instability turning to turbulent like fluctuations at the end of the process. However during a non negligible period of the evolution low wave numbers are observed and this corresponds to the local maximum in enstrophy and to the plateau in energy observed in the decrease curves of figure 6.

c) External turbulence

In the case of the synthetic turbulence forcing the situation is quite different. The time evolution of overall energy and enstrophy has similar shapes but on shorter time scales (50 rather than 100) and the low wave number spectrum content is more marked. The difference is even more obvious on the pressure fields that clearly show that a Crow perturbation is excited and the turbulent flow is structured by this large scale instability. The presence of this type of instability also explains the more rapid fall in the energy curves as it is known to be faster than elliptical instability.
One of the questions that arise from this simulation is the influence of the periodic boundary conditions in the longitudinal direction. In this case the large scale perturbation is forced to return to the initial values. This box effect can be the reason for maintaining a periodic structure in the flow provoking a global instability when convective instability could prevail. This is one of the motivations for conducting the same kind of simulations in a spatial evolution context which is well inside the capacities of the numerical code used.

7. Conclusion

This numerical study shows that a pair of longitudinal contra-rotating vortices is receptive to both large and small wavelength disturbances. If the large scale periodic Crow instability is imposed it implies a clear development of regular vortex oscillation leading to vortex rings that behave separately.
In the case of a white noise high wave number elliptical like instabilities are observed. Although transient low wave number components appear during the evolution the destruction process of the vortex is mainly controlled by short wave lengths and the unsteadiness is excited after a relative long time. The decrease of energy and enstrophy exhibits two different slopes. This shape is attributed to the effect of these large scale components at intermediate stages and the dominance of small scale ones at long time.
Forced by external turbulence a pair of longitudinal vortices presents a large wavelength unsteadiness that develops at the beginning in the same way as Crow instability. It is also destructed in the final stage by small scale motion. A periodic structure is observed in the visualization during the transient that can be associated with the type of simulation conducted. Discussing this particularity opens the way to spatially evolving simulation that should provide another picture of the phenomenon.

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8. References

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